Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

16 May 2025

Zone A morning | **Zone B** morning | **Zone C** morning

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 18]

Cathie is a financial analyst studying the growth of two investment accounts, Account 1 and Account 2, for a new client.

Account 1 has an initial amount of 5000 US Dollars (USD). Interest is added to the amount in Account 1 at the end of each year in the following manner: $200\,\text{USD}$ at the end of the first year, $260\,\text{USD}$ at the end of the second year, $320\,\text{USD}$ at the end of the third year, $380\,\text{USD}$ at the end of the fourth year and $440\,\text{USD}$ at the end of the fifth year.

Assume the amount of interest continues to increase each year so that it follows an arithmetic sequence.

(a) Find

(i) the common difference.

(ii) the amount of interest, in USD, added at the end of the 10th year. [3]

(b) Show that the amount of money in Account 1 after n years may be expressed as

$$5000+\frac{n}{2}(340+60n).$$ [3]

(c) Hence or otherwise, find the amount of money in Account 1 at the end of 10 years. [2]

Account 2 has the same initial amount of $5000\,\text{USD}$. Account 2 pays 6.5% interest compounded annually. The interest is added to the amount in the account at the end of each year.

The amount in Account 2 after n years can be expressed as $5000 \times B^n$ where $B \in \mathbb{R}$.

(d) (i) Write down the value of B.

(ii) Hence or otherwise, show that Account 1 will have more money than Account 2 at the end of 10 years. [4]

The client is interested in a longer-term investment. Cathie finds that it will take at least m complete years for the amount in Account 2 to exceed the amount in Account 1.

(e) Find the value of m. [3]

(f) Determine the total interest added to Account 2 at the end of m years. Give your answer correct to the nearest dollar. [3]

2. [Maximum mark: 17]

A company produces electronic components on a large scale. They carry out quality control tests to determine whether the components meet the company's standards.

Zaakir, the owner of the company, wants the quality control team to analyse the distribution of the weights of the components.

Based on historical data, the quality control team knows that the weights of the components follow a normal distribution with a mean of 2.5 grams and a standard deviation of 0.15 grams.

(a) Find the probability that the weight of a component selected at random is greater than 2.8 grams. [2]

The probability that the weight of a component selected at random is greater than w grams is 0.8.

(b) (i) Sketch a diagram of a normal curve to show the area represented by this probability.

(ii) Find the value of w. [4]

To pass Test 1, the weight of a component must be between 2.3 grams and 2.7 grams.

(c) (i) Find the probability that a randomly selected component passes Test 1.

(ii) Find the expected number of components in a box of 200 that will pass Test 1. [3]

Zaakir asks the quality control team to conduct a more in-depth analysis by performing a new test, Test 2. The probability of a component passing Test 2 is 0.95. The team randomly selects one box and tests each of the 200 components.

(d) Find the probability that exactly 190 components pass Test 2. [2]

(e) Find the probability that at least 188 components pass Test 2. [2]

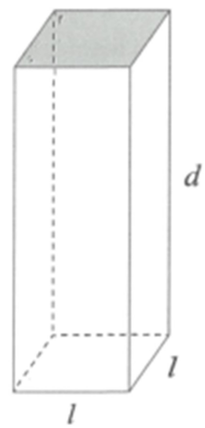
Instead of testing all 200 components, Zaakir now decides to test a random sample of 12 components from a box of 200 components. He decides that the box will only be dispatched if at least 10 of the 12 components pass both Test 1 and Test 2. The results of Test 1 and Test 2 are independent.

(f) Find the probability that the box is dispatched. [4]

3. [Maximum mark: 20]

Kailash manufactures drink containers in the shape of a cuboid. The container has a square top and a square base of length, $l\,\text{cm}$. Its height, $d\,\text{cm}$, is three times the length of the base.

diagram not to scale



(a) Write down an expression for d in terms of l. [1]

The container can hold $375\,\text{cm}^3$ of drink.

(b) Find the value of l and d. [3]

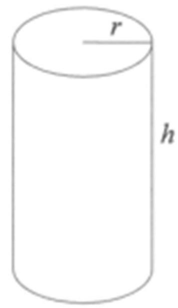
(c) Calculate the total external surface area of the container. [3]

(This question continues on the following page)

(Question 3 continued)

To reduce environmental impact, Kailash is trying to minimize the amount of material needed for the production of the $375\,\text{cm}^3$ container.

He is willing to change the shape to a cylinder with radius $r\,\text{cm}$, and height $h\,\text{cm}$, as shown below.



The cylindrical container of drink must also hold $375\,\text{cm}^3$.

(d) Find an expression for the height, h, of the container in terms of r. [2]

Let the total external surface area be $A\,\text{cm}^2$.

(e) Show that $A = 2\pi r^2 + \dfrac{750}{r}$. [2]

(f) Find $\dfrac{\mathrm{d}A}{\mathrm{d}r}$. [3]

(g) Hence or otherwise

(i) find the value of r that will minimize A.

(ii) find the minimum value of A needed for the cylinder. [3]

To produce the containers, additional material is required:

- 10% additional surface area for the cuboid
- 25% additional surface area for the cylinder.

Kailash will choose the container that requires the least total amount of material.

(h) Determine which container Kailash should choose. Justify your answer. [3]

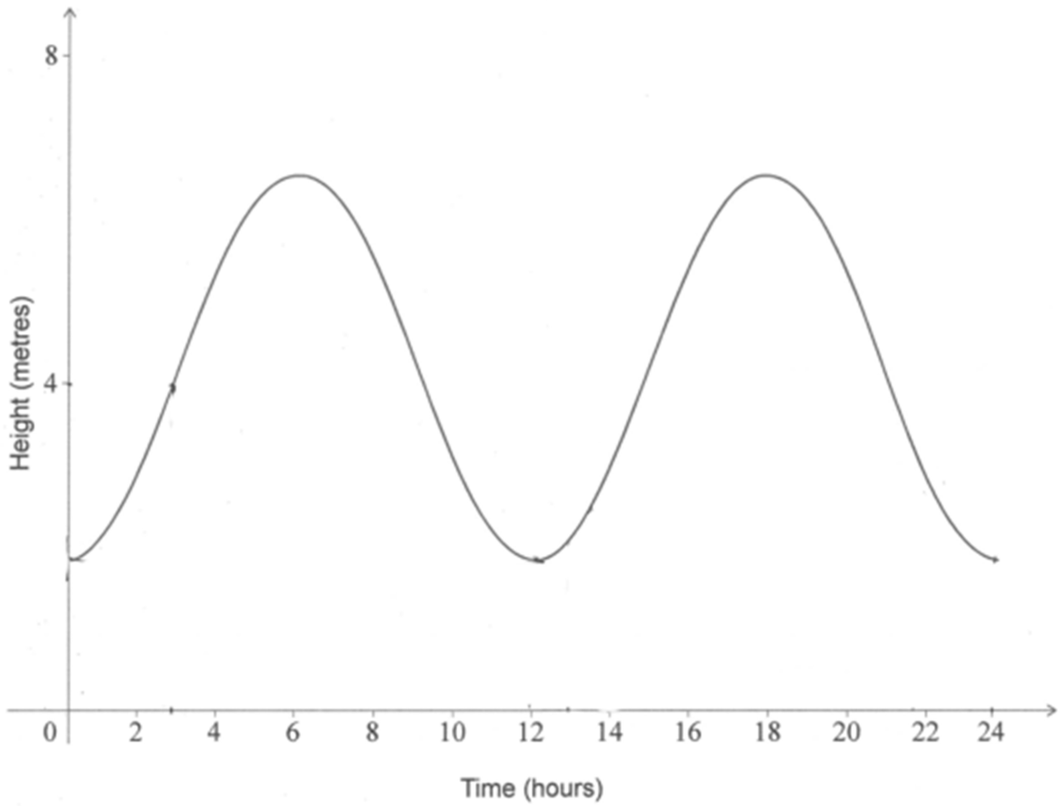
4. [Maximum mark: 13]

On a particular day the height of the tide, h, in metres, at Albion harbour can be modelled by the function

$$h(t) = -2.5\cos(bt°) + 4.5, \text{ where } b \in \mathbb{R}, 0 \le t \le 24$$

and t represents the number of hours after midnight.

The graph of h is shown in the following diagram.



(a) Show that the value of b is 30. [1]

(b) Find the height of the tide when $t = 5$. [2]

(c) Write down

(i) the amplitude of h.

(ii) the equation of the principal axis. [3]

(This question continues on the following page)

(Question 4 continued)

Boats can only leave or return to Albion harbour when $h(t) \geq 2.65$. Robin wants to leave the harbour to go fishing as soon as possible after the time is 12:00.

(d) Determine the earliest possible time that Robin could leave the harbour. Give your answer to the nearest minute. [3]

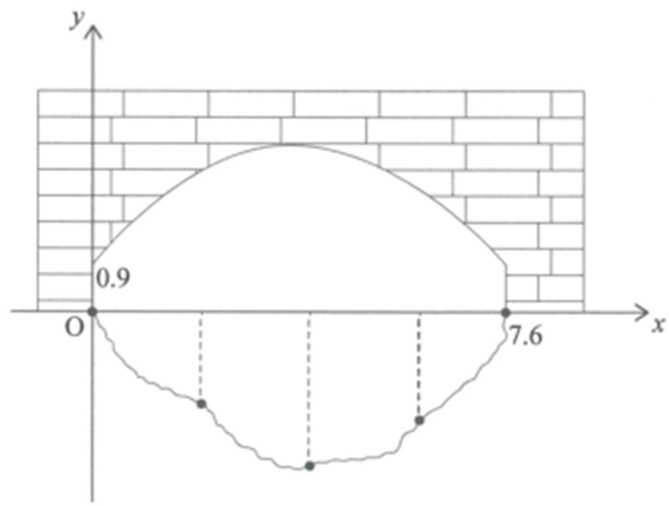
The boat will take 15 minutes to travel from the harbour to the fishing site. Robin intends to return to the harbour on the same day.

(e) Determine the maximum length of time he could spend at the fishing site, in hours, and still be certain he will be able to enter the harbour on his return. [4]

5. [Maximum mark: 12]

The diagram shows the cross-section of a bridge and a river. A coordinate system has been added with the origin, O, at the point where the bridge meets the water on one side. All units are in metres.

diagram not to scale



A researcher wants to calculate the volume of water that flows under the bridge. To do this he takes measurements of the depth every $1.9\,\text{m}$ from O. The depths are shown in the following table.

Horizontal distance from O in metres	0	1.9	3.8	5.7	7.6
Vertical depth of water in metres	0	1.68	2.81	2.32	0

(a) Use the trapezoidal rule to find the cross-sectional area of the river as it passes under the bridge. [3]

The water flows under the bridge at a rate of $0.3\,\text{m}\,\text{s}^{-1}$.

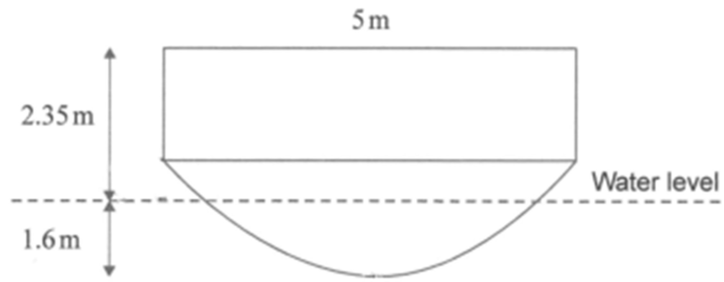
(b) Find the volume of water that passes under the bridge each second. [2]

(This question continues on the following page)

(Question 5 continued)

A boat is travelling along the river. The cross-section of the boat and the water level is shown in the following diagram.

The top of the boat is parallel to the water level and has a width of 5 m. The height of the boat is 2.35 m above the water level and the lowest part of the boat is 1.6 m below the water level.



The boat is travelling down the centre of the river.

(c) Find the vertical distance between the lowest part of the boat and the bottom of the river as it passes under the bridge. [1]

The curved arch of the bridge can be modelled by the equation

$$y = -0.15x^2 + 1.14x + 0.9,\ 0 \le x \le 7.6.$$

(d) Find the maximum height of the curved arch above the water level. [2]

(e) Determine whether the top of the boat will be able to pass under the bridge. [4]